Chapter 66 Dual-Update Rate INS Aided Carrier Phase Lock Loop for New Generation Global Navigation Satellite Signals

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Abstract Most of the new generation of satellite navigation signals using a pilot channel to improve the tracking capability of the receiver. Without constraints of navigation data bit length, the carrier phase-locked loop (PLL) can prolong easily the Pre-detection integration time (PIT) to improve the anti-jamming ability of the receiver, but the prolongation of the PIT is still subject to the restrictions of the carrier dynamic conditions. In this paper, an algorithm of dual-update rate inertial navigation system (INS) aided carrier PLL was proposed. First, the mathematic model of INS aided carrier PLL was developed with the Kalman filter. Taking into account the high INS update rate and high short-term accuracy, the INS observation information was used to control the frequency of the carrier numerically controlled oscillator (NCO) during the PLL integral period. At the end of one integral period, the output of phase discriminator was used to correct the reproduced carrier phase. The algorithm proposed by this paper can effectively prolong the PLL PIT in high dynamic conditions and improve the anti-jamming ability of the receiver in high dynamic conditions. Finally, the effectiveness of the proposed algorithm was verified by simulation.

Keywords Pilot channel · INS aiding · PLL · High dynamic · Anti-jamming

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66.1 Introduction

Carrier PLL is used to track the Doppler frequency of the received satellite carrier signal, and is the key technology of the global navigation satellite system (GNSS) receiver. Narrower noise bandwidth of the carrier tracking loop and longer PIT is required in order to improve the anti-jamming ability of the receiver, but prolongation of the PIT is limited by the bit length of the navigation data and the carrier dynamic. Most of the new generation of navigation satellite signals, such as the GPS L2, L5 signals, the Galileo E1, E5, E6 signals and the Compass B1, B2, B3 signals, use or plan to use a pilot channel in addition to the traditional data channel [[1,](#page-9-0) [2](#page-9-0)]. The pilot channel can use a pure PLL which avoids squaring loss and has a larger linearity region than a Costas loop [\[3](#page-9-0)]. What is more, when tracking pilot signal the PIT can be prolonged without considering bit boundaries.

In some applications, the receiver is required working in the high dynamic and heavy jamming conditions. If there is a large accelerated motion between the receiver and the satellite, long PIT will cause for a large dynamic stress error of the PLL, even cause for the PLL loss of lock. To solve this problem, the literatures Yao et al. [\[4](#page-9-0)] and Jin et al. [\[5](#page-9-0)] proposed respectively dual-update rate PLL algorithm and dual-update rate frequency assisted phase lock algorithm. But the tracking ability of these methods depend largely on the tracking loop which has the higher update rate. In the weak signal and high dynamic conditions, once the high update rate loop loses lock, it will be difficult to re-capture the signal.

INS and GNSS are different types of navigation systems. The former has higher update rate and higher short-term accuracy. Once the GNSS PLL loses lock temporarily for some reasons, the output information of INS can be used to re-capture the GNSS signal quickly. The literatures Alban et al. [\[6](#page-9-0)] and Babu and Wang [[7\]](#page-9-0) have researched the algorithms of INS aiding PLL and made detailed performance analysis. However, these algorithms fuse the INS observational data and the PLL phase discriminator in the same update rate. When the long PIT is used, if the acceleration between the receiver and the satellite is large, the phase error between the received carrier and the recovered carrier may exceed the linear range of the phase discriminator, thereby causing for the PLL phase gliding or even loss of lock [\[8](#page-9-0)]. In this paper, take into account of the high update rate and high short-term accuracy of INS [\[9](#page-9-0)], a new algorithm was proposed. Assuming that the update rate of INS is higher than the update rate of PLL discriminator, the INS observation information was used to control the frequency of the carrier NCO during the coherent integral period. At the end of one integral period, the output of phase discriminator was used to correct the recovered carrier phase. Finally, the effectiveness of the proposed algorithm was verified by simulation.

66.2 Basic Model of INS Aided PLL

66.2.1 Signal Mathematic Model

The received signals in the present of thermal noise used in this process can be modelled as [\[4](#page-9-0)]:

$$
s(t) = \sqrt{2C/N_0T}x(t-\tau)\cos((\omega_{IF} + \omega_D)t + \theta) + n(t)
$$
 (66.1)

where C is the received power, N_0 is the thermal noise power spectral density, T is the coherent integral period, $x(t)$ is the pseudo-random noise (PRN) codes modulated on the pilot channels, τ is the code group delay due to travel time, ω_{IF} is the intermediate carrier frequency of the sending signal, ω_D is the Doppler shift frequency, θ is the carrier phase of the signal and $n(t)$ is the normalized thermal noise. In the baseband process, $s(t)$ is multiplied by an in-phase replica and a quadrature-phase replica of the estimated carrier, respectively, and then the signals are correlated with a replica of PRN code over a period T referred to as the coherent integral time. The correlator outputs can be modeled as:

$$
I_{k} = A_{k} \cos(\Delta \omega_{k} T/2 + \Delta \theta_{k}) + n_{Ik}
$$
 (66.2a)

$$
Q_{k} = A_{k} \sin(\Delta \omega_{k} T/2 + \Delta \theta_{k}) + n_{Qk}
$$
 (66.2b)

where $A_k = \sqrt{2C/N_0TR(\Delta \tau)} \sin c(\Delta \omega_k T/2)$, $\text{sinc}(x) = \sin x/x$, $R(x)$ is the autocorrelation function of PRN code, $\hat{\tau}$ is the estimated code delay time, $\Delta \tau$ = $\tau - \hat{\tau}$, $\omega_{k|k-1}$ and $\theta_{k|k-1}$ are predicted carrier Doppler and phase at the $(k - 1)$ th interval, respectively, $\Delta \omega_k = \omega_k - \omega_{k|k-1}$, $\Delta \theta_k = \theta_k - \theta_{k|k-1}$, n_{Ik} and n_{Ok} are I and Q baseband noises in the pilot channel respectively. It is assumed the tracking loop is locked. At this condition, $\Delta \tau$ and $\Delta \omega_k$ is approach to 0. Thus, $R(\Delta \tau)$ sin $c(\Delta \omega_k T/2)$ is approach to 1 and Eqs. (66.2a, b) can be rewritten as:

$$
I_{k} = \sqrt{2C/N_{0}T} \cos(\Delta\omega_{k}T/2 + \Delta\theta_{k}) + n_{Ik}
$$
 (66.3a)

$$
Q_{k} = \sqrt{2C/N_{0}T} \sin(\Delta \omega_{k}T/2 + \Delta \theta_{k}) + n_{Qk}
$$
 (66.3b)

Since there is no message data bit modulated on pilot channel carrier, one can use a so-called coherent discriminator [[4\]](#page-9-0), the expression of which is:

$$
r_{\theta,k} = \frac{Q_k}{\sqrt{2C/N_0T}}\tag{66.4}
$$

Assuming $\Delta \tau$ and $\Delta \omega_k$ is approach to 0, substituting Eq. (66.3b) into Eq. (66.4), we have

$$
r_{\theta,k} = \Delta \theta_k + \Delta \omega_k T / 2 + \delta \theta_k \tag{66.5}
$$

where

$$
\delta \theta_k = \frac{n_{Q,k}}{\sqrt{2C/N_0 T}} \approx \frac{n_{Q,k}}{\sqrt{\frac{1}{N} \sum_{i=1}^{N-1} (I_{k-i}^2 + Q_{k-i}^2)}}
$$
(66.6)

N is the smoothing times

66.2.2 Kalman Filter Model of INS Aided PLL

Take the received carrier phase $(\theta(t))$, Doppler frequency $(\omega(t))$ and the Doppler frequency variation rate $(a(t))$ as state variables. It is assumed that the three order derivative of the carrier phase is random white noise witch main dues to carrier's dynamic and the clock oscillator. The dynamic equation is:

$$
\begin{bmatrix}\n\dot{\theta}(t) \\
\dot{\omega}(t) \\
\dot{a}(t)\n\end{bmatrix} = \begin{bmatrix}\n0 & 1 & 0 \\
0 & 0 & 1 \\
0 & 0 & 0\n\end{bmatrix} \begin{bmatrix}\n\theta(t) \\
\omega(t) \\
a(t)\n\end{bmatrix} + \begin{bmatrix}\n0 \\
0 \\
1\n\end{bmatrix} j(t)
$$
\n(66.7)

Discretizing Eq. (66.7), we have:

$$
\mathbf{x}_{k+1} = \mathbf{\Phi} \mathbf{x}_k + \mathbf{w}_k \tag{66.8}
$$

where $\mathbf{x}_k = [\theta_k, \omega_k, a_k]^T$,

$$
\mathbf{\Phi} = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}
$$
 (66.9)

 \mathbf{w}_k is Gaussian white noise which covariance matrix is $\mathbf{Q}_k = E[\mathbf{w}_k \mathbf{w}_k]^T$.

There are two observables for the INS-aiding PLL. One is the output of coherent correlator, and the other comes from the velocity of INS. Thus, the observation equation can be written as

$$
\mathbf{z}_{k+1} = \begin{bmatrix} \theta_{k+1} + \omega_{k+1} T/2 + \delta \theta_{k+1} \\ \omega_{k+1} + \frac{2\pi f_c}{c} \delta v_{k+1} \end{bmatrix} = \mathbf{H} \mathbf{x}_{k+1} + \mathbf{w}_{k+1}
$$
(66.10)

where δv_{k+1} is INS speed error projected in the sight of receiver and satellite, c is light velocity, f_c is radio carrier frequency. The observation matrix is:

$$
\mathbf{H} = \begin{bmatrix} 1 & T/2 & 0 \\ 0 & 1 & 0 \end{bmatrix} \tag{66.11}
$$

The observation covariance matrix is:

$$
\mathbf{R}_{k+1} = \begin{bmatrix} \sigma_{r,k+1}^2 & 0\\ 0 & \sigma_{INS,k+1}^2 \end{bmatrix}
$$
 (66.12)

where $\sigma_{r,k+1}^2 = \frac{1}{2CN_0T}$, $\sigma_{INS,k+1}^2 = \frac{4\pi^2f_c^2}{c^2}\sigma_{v,k+1}^2$, $\sigma_{v,k+1}^2$ is the variance of δv_{k+1} . The Kalman state update equation of carrier is:

$$
\mathbf{x}_{k+1|k+1} = \mathbf{x}_{k+1|k} + \mathbf{K}_k (\mathbf{z}_{k+1} - \mathbf{H} \mathbf{x}_{k+1|k})
$$
 (66.13)

where \mathbf{K}_k is the gain matrix of Kalman filter. As can be seen from the Eq. (66.13), the first part of the innovation is:

$$
y_1 = (\theta_{k+1} + \omega_{k+1}T/2 + \delta\theta_{k+1}) - (\theta_{k+1|k} + \omega_{k+1|k}T/2)
$$

= $\Delta\theta_k + \Delta\omega_kT/2 + \delta\theta_k$ (66.14)

Comparing Eq. (66.14) with Eq. ([66.5](#page-2-0)), it shows that y_1 is the output of the phase discriminator [\[10](#page-9-0)].

66.3 Proposed Dual Update-Rate INS Aided PLL

The base Kalman filter model described in [Sect. 66.2](#page-2-0) requires both observables have the same update rate. Under weak signal conditions, the PIT is often prolonged to improve the anti-jamming ability of PLL. In the above model, the carrier NCO control frequency is a fixed constant. If the acceleration between the receiver and the satellite is too large, the phase error between the received signal and the numerically controlled oscillator (NCO) may exceed the linear range of the phase discriminator, thereby causing for the PLL phase gliding or even loss of lock.

Taking into account the INS's high update rate and high short-term accuracy, the output of INS can be used to update the NCO control frequency in the PLL PIT. Figure [66.1](#page-5-0) is the schematic diagram. In the diagram, $t1 \sim t6$ is assumed the update time of INS, t1 and t6 is the update time of PLL discriminator. $\Delta\theta$ is the difference between received carrier phase and local generated carrier phase without INS aiding. $\Delta\theta'$ is the difference when there is INS aiding during PLL coherent integral period. It's obvious that $\Delta\theta'$ is smaller than $\Delta\theta$. Therefore, the PLL loop can withstand larger carrier dynamic. Based on this, we propose a dual update rate INS aided PLL algorithm.

Assuming the carrier PIT is M times of the INS update time. The observation matrix with INS separately providing observations is $H' = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$. The Eq. [\(66.9\)](#page-3-0) can be rewritten

$$
\Phi' = \begin{bmatrix} 1 & T/M & T^2/2M^2 \\ 0 & 1 & T/M \\ 0 & 0 & 1 \end{bmatrix}
$$
 (66.15)

The rank of the observability matrix can be calculated as [\[11](#page-9-0)]:

$$
Rank\left(\begin{bmatrix}H'\\H'\Phi'\\H'\Phi'^2\end{bmatrix}\right) = 2 < 3
$$
\n(66.16)

Equation (66.16) shows that the state vector \mathbf{x}_k is not completely observable. Further analysis shows that the phase is not observable. During PLL coherent integral period, INS output is the only observations. Selecting $\mathbf{x}_{k}^{INS} = [\omega_k^{INS}, a_k^{INS}]^T$ as the state vector, state equation and observation equation are:

$$
\mathbf{x}_{k+1}^{INS} = \mathbf{\Phi}^{INS} \mathbf{x}_k^{INS} + \mathbf{w}_k^{INS}
$$
 (66.17)

$$
\mathbf{z}_{k+1}^{INS} = \mathbf{H}^{INS} \mathbf{x}_{k+1}^{INS} + \mathbf{v}_{k+1}^{INS}
$$
 (66.18)

where

$$
\mathbf{\Phi}^{INS} = \begin{bmatrix} 1 & T/M \\ 0 & 1 \end{bmatrix} \tag{66.19}
$$

 \mathbf{w}_k^{INS} and \mathbf{v}_{k+1}^{INS} are the state noise and observation noise respectively, and the covariance matrices are respectively:

$$
\mathbf{Q}_k^{INS} = E[\mathbf{w}_k^{INS}(\mathbf{w}_k^{INS})^T] \tag{66.20}
$$

$$
\mathbf{R}_{k+1}^{INS} = E[\mathbf{v}_{k+1}^{INS}(\mathbf{v}_{k+1}^{INS})^T] = \sigma_{INS, k+1}^2
$$
 (66.21)

Owned to INS aiding, the NCO control frequency control frequency is no longer kept constant within a PLL coherent integral period. Therefore Eq. (66.8) (66.8) (66.8) can be re-expressed as:

$$
\begin{bmatrix}\n\theta_{k+M}^{PL} \\
\omega_{k+M}^{PL} \\
\alpha_{k+M}^{PL}\n\end{bmatrix} = \Phi \begin{bmatrix}\n\theta_k^{PLL} \\
\omega_k^{PLL} \\
\alpha_k^{PLL}\n\end{bmatrix} + \begin{bmatrix}\n\frac{T}{M} \sum_{i=0}^{M-1} \omega_{k+i}^{INS} \\
0 \\
0 \\
0\n\end{bmatrix} + \mathbf{u}_k^{PLL} + \begin{bmatrix}\n\frac{T}{M} \sum_{i=0}^{M-1} n_{\omega,k+i}^{INS} \\
0 \\
0 \\
0\n\end{bmatrix}
$$
\n(66.22)

where θ_k^{PLL} is the carrier phase, ω_k^{PLL} and a_k^{PLL} are residual carrier frequency and acceleration, $n_{\omega,k}^{INS}$ is error noise of INS which variance can be calculated by high update rate Kalman filter.

With INS aiding during coherent integral period, the PLL's discriminator is:

$$
r_{\theta, k+M} = \frac{\sum_{i=0}^{M-1} Q_{k+i}}{\sqrt{2(C/N_0)TM}}
$$
(66.23)

Neglecting INS's error, Eq. (66.23) can be rewritten:

$$
r_{\theta,k+M} \approx \Delta \theta_k^{PLL} + \frac{T \omega_k^{PLL}}{2} + \frac{\sum_{i=0}^{M-1} n_{Q,k+i}}{\sqrt{2(C/N_0)TM}}
$$
(66.24)

PLL observation noise matrix is:

$$
R_k^{PLL} = \frac{M}{2(C/N_0)T}
$$
 (66.25)

66.4 Simulation Results

Assuming the GNSS pilot channel frequency is 1575.42 MHz, the INS update rate is 200 Hz (5 ms), the speed accuracy of INS after Integrating with GNSS is 0.2 m/s. The input pilot channel signal is generated by Matlab. Figure [66.2](#page-7-0) shows the pilot channel Doppler frequency shift. In the first second, the velocity between the satellite and receiver is assumed to be 200 m/s. From 1 to 3 s the acceleration between them is assumed to be 50 g. Then the acceleration is assumed to be 0. And now the velocity between the satellite and receiver keeps 1,180 m/s. Other dynamic (including the oscillator's error) is assumed to noise, and the standard deviation is assumed to be 0.5 g/s . A phase locked indicator is used to indicator the tracking accuracy degree of the carrier phase. The phase locked indicator is:

$$
PLI = \frac{I^2 - Q^2}{I^2 + Q^2} \approx \cos(2\Delta\theta)
$$
 (66.26)

When the PLL is locked, the PLI is approach to 1.

First, we simulate the tracking capability of new algorithm under high dynamic condition. Figure [66.3](#page-7-0) shows the tracked Doppler frequency when the carrierto-noise ratio is $43 dB * Hz$. When the PLL PIT is 5 ms, PLL can correctly track the Doppler frequency. When the PIT is prolonged to 10 ms, PLL loses lock. However, the new dual-update INS aided PLL algorithm can track the Doppler frequency even the PIT is prolonged to 20 ms.

Fig. 66.2 Doppler frequency caused by vehicle dynamic

Followed, we simulate the tracking capability of new algorithm in the low carrier-to-noise ratio and high dynamic conditions. Figure [66.4](#page-8-0) shows the output of the phase locked indicator when the carrier-to-noise ratio is $26 dB * Hz$. When the PIT is 5 ms, the alone PLL can generally track the carrier, but there is phase glide at 1 and 3 s when large acceleration between the receiver and satellite appears and disappears. The new dual-update INS aided PLL algorithm can track the carrier without any phase glide even when the PIT is prolonged to 20 ms. And it is obvious that the accuracy degree of phase tracked when the PIT is 20 ms is higher than when PIT is 10 and 5 ms.

Figure [66.5](#page-8-0) is the curve of phase jitter using proposed algorithm to track the carrier dynamic process of Fig. 66.2 in different carrier-to-noise ratio conditions. For each carrier-to-noise ratio, 50 Monte Carlo simulations was made. In the low carrier-to-noise ratio and high dynamic conditions, the PLL coherent integral time can be significantly prolonged using the proposed algorithm, and a greater spreading gain can be obtained to improve the anti-jamming ability of the receiver.

66.5 Conclusion

New generation navigation signals have pilot channels which improve the antijamming ability of the receiver, but when in the high dynamic condition this advantage has been very limited. In this paper, the dual-update rate INS aided GNSS PLL algorithm was proposed. When the PLL loop coherent integral time is longer than INS update time, the INS observation information was used to control the frequency of the carrier NCO during the coherent integral period. At the end of the coherent integral period, the phase discriminator output was used to correct the carrier phase. The new algorithm can reduce the limitation of high dynamic on the PLL coherent integral time. In the low carrier-to-noise ratio and high dynamic conditions, the PLL coherent integral time can be significantly prolonged using the proposed algorithm, and a greater spreading gain can be obtained to improve the anti-jamming ability of the receiver.

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